Examiners' Report

Principal Examiner Feedback

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Pearson Edexcel International A Level

In Pure Mathematics (WMA12/01)

General

The paper was well received overall, with the early questions providing access at all grade boundaries. However, the final three questions did prove very challenging, though there was no indication that time was a problem. The topic of proof continues to test even the best of candidates, and there were still signs of under-preparedness on some topics, possibly due to the ongoing pandemic.

Report on individual questions

Question 1

For the well-prepared candidate, this was a good starter question with 75% able to score more than half marks, though only a third scored full marks. The mean mark was 4/7.

In part (a) the trapezium rule was well known and terms were correctly placed within the bracket structure. The majority correctly used $h=\frac{1}{2}$ but a few went for 0.4 confusing the given 5 ordinates with the number of strips needed.

Common errors were failing to multiply the whole bracket structure by $\frac{1}{2}h$ and slips when using their calculator. Repetition of values was less common that in previous series.

Most gave the expected answer of 11.79, often following 11.78725, the full value from their calculator confirming that they had done the calculation correctly. There did appear to be more responses using separate trapezia than in previous series: these were usually successful.

A few candidates used the integral button on their calculator to give the answer 11.78 with no supporting working: these received no marks as the question was clear that the trapezium rule must be used.

In part (b), there was an expectation to see some working to justify how the answer to (a) was being used, as the question specified the method should be made clear. Candidates need to heed such instructions to ensure they gain marks. Those using the calculator integral button and giving answers of 23.56 and 17.78 with no method, were not credited any marks, though this was relatively uncommon. Some chose to use the trapezium rule again which also scored zero since use of the answer to part (a) was required.

(i) Some candidates identified the multiple of 2 but struggled to understand how to apply this to the integral. Frequently the 2 was integrated giving rise to $(14-10)=4$ and either added to the answer to (a) or used as a multiplier. The follow through allowed for the accuracy mark meant that even those who had an incorrect attempt at part (a) were still able to gain credit in this part.

(ii) This part was better attempted than (i) but still challenging. A common error was to add 3 rather than integrating it. However, many spotted the solution to part (b) (ii) was obtained by simply adding 6 to their part (a) answer.

For a question with a very specific technique a surprising number of blank scripts were seen by some markers, perhaps suggesting that some centres had not taught this. Over 15% score zero marks for this opening question.

Question 2

Overall, this question was accessible with over 40% of candidates scoring full marks and a mean mark of 5.6 out of 8. It was a topic that seemed very familiar to candidates and less than 7% scored fewer than 2 marks (usually at least the marks in part (a) being scored).

In part (a), candidates needed to find the first derivative and the vast majority were able to do so correctly. A few made errors in reducing the powers by one.

In part (b), most candidates demonstrated they correct method to set their derivative equal to zero in order to find the stationary point. Although most gave fully correct algebraic steps to find the required x coordinate, others found the manipulation of the two fractional indices challenging. The most successful method saw the terms in the equation multiplied through by a common factor, usually $x^{½}$, followed by rearrangement to $x = 9$. A few tried a substitution such as $a = x^{1/2}$ and often went wrong at the final stage of undoing this substitution.

Occasionally, because of the way the equation was rearranged, an extra value of $x = 0$ was obtained which could not satisfy the original equation but was not penalised.

Many attempted a correct method for finding the y coordinate, but some did forget to do this entirely, while others erroneously substituted their x into their derivative. Care is needed to make sure that candidates know what a question requires.

In part (c) the idea of using the second derivative to determine the nature of the point was well known. Most solutions took the second derivative and attempted to find its value at the point found in (b). Some found the second derivative accurately but failed to do anything with it and so scored no marks. Others set it equal to 0 and solved again gaining no marks. It was rare to see an argument based on the reasoning that since the x coordinate from (b) was positive then both terms in the second derivative were negative. Of similar rarity was deducing an incorrect nature of the turning point from the correct negative second derivative.

Question 3

This binomial expansion question was accessible to almost all students, especially part (a) with part (b) more demanding. The means mark of 4/7 was driven mainly by part (a). One in 10 gained zero marks in total either via non-attempts or very poor attempts.

Part (a) saw the vast majority of candidates able to use the formula for a binomial expansion, usually using $\begin{pmatrix} n \\ r \end{pmatrix}$ notation although ${}^{n}C_{r}$ was also seen, or sometimes directly giving a numerical form. Around half the students seen gained all four marks for a fully correct binomial expansion. Where marks were dropped this was generally due to sign errors, using $\left(\frac{kx}{4}\right)$ instead of $\left(-\frac{kx}{4}\right)$ or errors in multiplying out brackets squared or cubed, in particular forgetting to square/cube the constant k.

Students found part (b) much more challenging, with a significant number of candidates making little or no attempt. For those who did attempt it, errors included finding only one x term rather than two (very common), incorrectly adding –1280k and –768 to obtain –2048k, omitting one or both minus signs, and failing to interpret the phrase 'the constant term is 3 times the coefficient of x' correctly, the latter usually resulting in the wrong side of the equation being multiplied by 3. Those who, due to a sign error, had obtained all positive terms in part (a) were still able to gain the two method marks in part (b).

Question 4

This five-mark question involving logarithms was attempted by most candidates and the majority answered it well with a good understanding of the laws of logarithms, with two thirds scoring at least 4 of the 5 marks, and a mean score of 3.2. Of the 15% or so who scored zero marks it was mainly due to not demonstrating any correct log work rather than not attempting at all.

Most candidates achieved at least the first mark, awarded for any correct use of the power law within the equation. Most achieved this mark by rewriting the $2 \log_3 (1 - x)$ term as $\log_3 (1 - x)^2$. Some expressed the 3 in the equation as $\log_3 27$ which also achieved this mark. Mistakes were made converting 3 to $\log_3 27$, with $\log_3 3$ or $\log_3 9$ seen.

Many candidates then combined two of the log terms correctly and formed a correct equation with the logarithms removed. However, the main errors in this question were seen in attempts to combine two terms. Not uncommon was just removing the logs from 3 terms, having converted 3 to $\log_3 27$ or not, thus getting $32 - 12x = (1 - x)^2 + 27$.

Less common was writing $\log(32 - 12x)/\log(1 - x)^2 = 3$ although some then recovered with the correct equation of $(32 - 12x)/ (1 - x)^2 = 3^3$. Occasionally errors such as expanding $\log(32 - 12x)$ to $\log 32 - \log 12x$ were also seen.

A few candidates combined the log terms correctly but thought $3^3 = 9$ though in some cases had already produced a correct equation for the first A mark.

Most proceeded to find a three-term quadratic equation and solve it to find the two values of x as $\frac{5}{3}$ and $-\frac{1}{9}$. However, many candidates lost the final mark by leaving both values of x obtained and not considering whether both values were valid. Another common response was to reject the correct solution of $x = -\frac{1}{9}$ just because it was negative, not considering the form of the given equation. While over 65% were able to score the first 4 marks, only a little over 25% scored all five.

Question 5

With a mean score of 5.3 out of 8 and 60% scoring over 5 marks this proved another good source of marks for candidates. Over 40% scored full marks, though 12% were unable to score any.

In part (a) most students were able to apply the remainder theorem to set up two equations in A, B and k and the majority correctly setting $\mathrm{f}(1) = k$ and $\mathrm{f}(-1) = -10k$. Most were then able to eliminate k.

Several variations of method were seen, the most common being to multiply $A + B - 7 = k$ by 10 and adding the result to $A - B - 13 = -10k$ or to substitute $A - B - 13 = -10(A + B - 7)$. A common error was to omit k completely (which scored no marks in this part), setting f(1) = 0 and f(-1) = 0 and getting nowhere. For those who had achieved equations with k, trying to eliminate either A or B and ending up with an equation in k before grinding to a halt was fairly common, though a small number of candidates did succeed in finding both A and B in terms of k and substituting into $11A + 9B$ to achieve 83.

Some candidates attempted the more complicated method of long division, with varying degrees of success, but rarely successful. A correct proof with no errors was required to earn the final A mark. This part did prove a good discriminator between good and poor candidates.

Part (b) was answered well with responses often gaining full marks even when part (a) had not. Most set $\mathrm{f}\left(\frac{2}{3}\right) = 0$ with $\mathrm{f}\left(-\frac{2}{3}\right) = 0$ being seen only rarely, so it is god to see that candidates are now much more familiar with the factor theorem. Again a small number attempted long division with similar degree of success as in part (a), and only a few made no attempt at this part of the question at all.

Having attempted $\mathrm{f}\left(\frac{2}{3}\right) = 0$, the majority of students managed to reach an equation $4A + 6B = 82$ or equivalent and most realised that this needed to be combined with $11A + 9B = 83$ from part (a) to find values for A and B. Calculators were sometimes utilised here, but generally either elimination or substitution were used, and some very elegant correct solutions were seen. Errors included confusion with fractions and slips when substituting.

In part (c), many candidates were able to successfully determine the quadratic. Of these, long division was the most popular approach, with the remainder using the method of inspection/factorisation. Students who had not successfully determined correct values for *A* and *B* in part (b) were still able to pick up a method mark, although if their *A* and *B* came out as fractions, they often struggled to carry out long division correctly. However, there were also many who made no attempt at this part having successfully found *A* and *B* in part (b), which is surprising give the skills demonstrated in the earlier parts showed the candidates quite capable. Many of these perhaps missed that part (c) was asked and assumed finding *A* and *B* was the conclusion of the question.

Question 6

Parts of this question were accessible to most candidates and it proved another good discriminator. The ramping in difficulty of the paper was apparent from this point, with the mean score being 4/8, only half marks and the score distribution was fairly uniform across all scores. Many were able to score some marks in (a) and (b) but part (c) was less well attempted and provided a challenge to the most able candidates.

For part (a) there were a variety of ways of showing that the angle is 90º. The most common approach to obtain the Method mark was by finding the gradients of lines *PQ* and *QR*. Also common was to attempt Pythagoras theorem with the lengths *PQ*, *QR* and *PR*. A few instead substituted these values into the cosine rule to obtain the mark, a few others found the midpoint of *PR* and showed it was a diameter, while a much smaller number used trigonometry to find the acute angles to show that *PQR* is a right angled triangle (though such a method could not gain the Accuracy mark as rounded decimals were used), and there were occasional more esoteric methods attempted.

However, many who gained the Method mark did not complete the proof adequately and so did not achieve the Accuracy mark. Of those using gradients, the calculation of the product was often not shown or a conclusion not made about the angle (deducing the chords were perpendicular was not sufficient). For those using Pythagoras, a common failing was to not give the values 2000, 500 & 2500 for the squares of the lengths, to show the required result. In addition, many did not gain the Accuracy mark, regardless of their method, by not giving an adequate concluding statement.

It is generally good practice to end a "show that" question by repeating the result that is to be shown. Many finished with a statement that the lines *PQ* and *QR* are perpendicular, but the A mark required the statement that angle *PQR* = 90º.

In part (b)(i) most obtained the correct coordinates of the centre, though a few did make errors at this stage. In part (ii) again most candidates could get started, understanding the need to use Pythagoras' theorem to find the distance between *P* and *R* or between their centre and one of the given points. Some had already found this during part (a).

A few found the diameter but did not divide by 2 to get the radius and a small number found 25 but went on to take the square root, while other made errors with signs when applying Pythagoras theorem due to the negative coordinates.

Part (c) was the most challenging part. Many scored no or just 1 mark here.

The B mark was for either finding the coordinates of point *S* or finding the gradient of the tangent at *S*. Successful solutions used the property that the centre found in (b) was the mid-point of *QS*. The general method of finding the gradient of the line *QS* and then the gradient of the line perpendicular to this which was the required tangent gradient was well understood. Finding the coordinates of *S* was usually done by inspection using a vector approach, with vector $QS = 2 \times$ vector QC, where C is the centre of the circle.

Those that found both *S* and the tangent gradient usually went on to form a correct tangent equation and almost all of these gave the equation in the required form.

The small number who obtained the equation of the tangent at point *Q* and then attempted to perform a translation to the point *S* were generally unsuccessful.

Question 7

Overall, this question appeared to be quite accessible to many candidates but also with many blank or zero score scripts with a bimodal score distribution of modes 0 and 8 out of 8. However, over 50% were able to score over half the marks and a mean score of 4.4 out of 8 shows it was a little more accessible than question 6. In both part marks were sometimes lost through failing to provide the final answers to sufficient accuracy and candidates should ensure they are working with enough decimal places to allow for awrt 1dp.

In part (i) by far the most successful approach was using the identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$, however, lots of poor algebra was seen with $3\tan(2x-15)=0$ being common, also incorrectly using $\tan\theta = \frac{\cos\theta}{\sin\theta}$ and $\tan(2x-15)=3$. Predictably, the weakest candidates tried to "expand" the brackets or cancel them out altogether and also observed was the coefficient of 3 being applied like the log power law. A few opted to square both sides and apply the Pythagorean identity with varying degrees of success and sometimes ending up with additional solutions.

Of those who reached $\tan(2x-15)=k$ most used it efficiently with the principal value being achieved. The correct order of operations to find x was usually then seen.

The range of x, that is $-90 < x < 90$, was not always taken into account causing some candidates to not consider the negative solution: $-73.3°$ was often missed, or the second angle quoted was outside of the specified range, commonly 106.7.

Part (ii) was the more successful of the two parts overall. The majority of candidates correctly applied the identity $\sin^2\theta = 1 - \cos^2\theta$ with only a small number failing to simplify to a three term quadratic, though sometimes there were some manipulation errors in simplifying, usually with the constant term. Some omitted to apply inverse cos to the roots of their quadratic and so only score the first method mark, but this was not common. More common was the candidates who reached $\theta = 1.69$ but failed to find the second angle correctly, either not attempting it or $\theta = 1.69 + \pi$ was seen often for those who tried.

Most candidates were comfortable working in radians in part, but a minority worked in degrees first and then converted to radians at the end. However, a small number of these forgot to convert to radians at the end or perhaps hadn't read the question properly. Premature rounding resulted in a loss of accuracy for a very small minority of candidates.

Question 8

This proved the first of three discriminating questions at the end of the paper. The mean score was around 3.4 out of 9 with less than 10% scoring more than 7 marks, though about 65% were able to score in the 4-7-mark range, showing access was available in the earlier parts. Parts (c) and (d) of this question proved challenging conceptually even for many good candidates. About a fifth of candidates score zero marks.

Part (a) was well answered part with most candidates achieving the correct answer from a correct method. Errors included use of a common difference of +2 instead of −2 or applying an incorrect formula. Care was needed with bracketing, though if a correct formula was stated first, credit was given. It was rare to see candidates listing the terms, though this did happen occasionally. A number of candidates did attempt to treat the first term separately and form a series starting with $a = 98$, often leading to confusion with the indexing in this part.

Most candidates achieved both marks for part (b) using a correct summation formula for 20 terms. Again, the main error was in using +2 instead of −2 as the common difference. Again some use $a = 98$ as a starting term, though usually they remembered to add in the extra term at the end. However, some added in 100×20 instead who took this route to end up with an incorrect answer.

Part (c) is where many candidates failed to understand the context with many assuming they had to start the geometric series from the first term of $a = 100$ again. Previous examinations have seen questions with such set ups of alternative series for the same situation, so candidate may have been expecting similar, rather than one series taking over from the other, so seeing $100r^{21} = 60$ solved was common. Other candidates had an indexing error, setting up $62r = 60$. But those who did read the question carefully enough to understand the context usually answered correctly.

In part (d) very few candidates scored full marks, as noted above. The first method mark was accessible to all who attempted the part, even those who had misunderstood the context, for using a correct geometric summation with their r from part (c) and even allowing $a = 100$ for the misinterpretation.

Some students started with a geometric n^{th} term formula or an arithmetic summation and achieved no marks, but many were able to gain this first mark.

However, only a very small amount, maybe less than 5%, of candidates then failed to set up a valid equation. They were not able to correctly process the overlap of the 20th term of the arithmetic series with the first term of the geometric series, and double counted this term (62). They failed to realise that the first term in the geometric series should be $62 \times r$ (or alternatively that the 62 should be subtracted from 1620). Also some candidates used 3 instead of 3000 in the calculation, and many did not consider the arithmetic series part at all, just setting a geometric series sum to 3000.

The third method mark could be achieved without the second method mark being awarded but did require an understanding of the context allowing from the confusion over the repeated term. This required use of 3000 and their answer to (b) within the calculation and correctly using logs to achieve an answer for n. The manipulation of logarithms was generally good so those who set up a suitable equation usually score this mark, making 2/4 common in (d).

The final answer mark was for a fully correct solution and given most candidates didn't achieve the second method mark this accuracy mark was also very rarely awarded.

Question 9

Another very challenging question and the need for the use of a constant in the limit provided a challenge for many. One third of candidates scored no marks at all in this question, with another 10% scoring only one mark (usually the first M in (b)). Distribution across the other marks was evenly spread with a mean score of 3.5 out of 10.

For part (a) many candidates were perfectly able to set up the equation needed to solve for x as expected, however many lacked the required skill in solving a quadratic of this form where there is a coefficient that is non numerical so success in achieving the correct coordinates was more limited. Typical incorrect answers were 1 + m or getting to $x^2 + mx - x = 0$ and not knowing to factorise (or divide by x which is fine in this case as we know $x = 0$ is already known). A few candidates did not find the y coordinate after finding the x coordinate.

In part (b) nearly all candidates were able to achieve at least 1 mark and many 2 for integrating and substituting in their value of x form part (a). Of those who scored only one mark, it was nearly always the first one here. But some use limits of 0 and 1, with a reticence to apply a limit that is a variable apparent.

Many were aware that the area was given by integrating curve – line, with most using $\int (x - x^2) - (mx)\ \mathrm{d}x$ and were able to gain the 4th mark here (as long as suitable limits were used). A few candidates attempted line – curve instead, which could gain only the first 3 marks (if correct).

Even when candidates had the correct method many were not able to demonstrate sufficient algebra skill to prove the result with sufficient intermediate working, with the given answer often appearing from insufficiently simplified or incorrect work (e.g. when line – curve had been used) with no justification. A focus on working with factorisation with brackets would benefit candidates, as most typically preferred to expand brackets as opposed to factorise out the common factor $(1-m)$. This led to an incredible amount of wasted time by candidates and typically many errors in expanding so many brackets.

Part (c) was poorly answered with very few achieving a version of the correct answer. Many did not manage to think of a correct strategy and even among those that did many had once again expanded their brackets which was a long and cumbersome approach leading to many errors. There were a small number of very succinct and perfect solutions by the best candidates, which were pleasing to see. Candidates who found the area under the curve bounded by the x-axis and sets equal to 2 times $\frac{(1-m)^3}{6}$ were generally successful, but those who attempted to find the area of region 2 by integrating the curve with limits $1-m$ and 1, adding the area of the triangle and setting equal to $\frac{(1-m)^3}{6}$ usually were unable to solve their resulting equation successfully to an exact value.

<u>Question 10</u>

Overall candidates did not score well on this question with a mean score of little over 1 out of 5, with many not attempting part (ii). A third scored no marks (though few were completely blank, as (i) was usually attempted), with just under half scoring only 1 mark. Full mark responses to this question were rare as the attention to detail needed to fully complete the proof was widely lacking.

Part (i) was reasonably successfully undertaken, with the idea of a counter example gaining traction from previous exam series. Many gave a suitable example with $p = 7$ and $p = 13$ being the examples most commonly used. The mark scheme required a minimal conclusion following the calculation and so many obtained this mark by simply writing ‘not prime’ although some showed factors eg $15 = 5\times3$.

Of those who failed to score a few candidates used values of p which were not prime, often $p = 4$, while others did not seem to understand what they were being asked to prove and simply wrote, for example, $p = 3$ is prime and $2p + 1 = 7$ is also prime.

Part (ii) was not well approached, with many candidates leaving this part unanswered or thinking that they could answer this question by evaluating the given expression using different values of n and checking just a few cases always gave even result. Such approaches scored no marks.

However, there were also many who realised the requirement to consider the cases when n is odd and even, although many of these did not know the need to use $2k$ and $2k \pm 1$ to represent n and answer the question algebraically. Many of those that did use algebra in this way went on to obtain correct expressions in k for both odd and even cases. It was quite common for candidates to reach the quadratics and then state they were even without referring to individual terms or taking out a factor of two, which would lose the final A mark. Others did not obtain the final A mark because either they did not give a concluding statement or because they had used the variable n in their algebraic expressions for odd/even – eg used $2n + 1$ to represent an odd number.

There was a small proportion of candidates who attempted logical reasoning, attempting to prove the result by stating results for odd/even without using algebra (eg odd × odd = odd). This approach was often not well explained or comprehensive. Candidates who used this logic approach only achieved the first two Method marks at most as to gain further marks they would have needed to prove all the relevant properties for combining odd/even numbers. The assumption of, for instance, the product of two even numbers being even should not be assumed without proof at this level, and candidates should be encouraged to explore algebraic approaches to proof questions such as this one.

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